AP Calculus AB   Name _____
Chapter 5 Test Review, Circuit Style!   Date _____
                        No Calculator

How the **Circuit** works:

> ➢ Work problem #1. Find the problem with the answer to problem 1. Label this problem #2.
>   Continue numbering the problems in this fashion. You should get 25 and 50 as your last answer.

Answer: 25 and 50 # 1 – Evaluate: $\lim\limits_{x \to 0} \dfrac{\sqrt{x+1}-2}{x-3}$	Answer: $-2\sqrt{3}$ #_____ - At what value(s) of x does $f(x)=x^4-18x^2$ have a relative minimum?
Answer: $-\dfrac{2}{5}$ #_____ - Find the derivative, $\dfrac{dy}{dx}$, of $x^2y-5y=12$ and evaluate it at the point $(1,\,-3)$.	Answer: $12x^2 \sin^3\left(x^3\right)\cos\left(x^3\right)$ #_____ - Evaluate: $\lim\limits_{x \to \infty} \dfrac{5x^3-3}{2x^3+1}$
Answer: $(1,\,5)$ #_____ - A particle moves along a horizontal line so that its position is given by $s(t)=t^3-9t^2+15t+4$, where $t \ge 0$. When is the particle speeding up?	Answer: $\dfrac{1}{3}$ #_____ - Find y' given $y=x^2 \cos x$.

Answer: $-x^2 \sin x + 2x \cos x$

#_____ - Find $f'(x)$ given $f(x) = \dfrac{x^2}{\tan x}$.

Answer: 2

#_____ - Write an equation of the line tangent to the graph of $y = x^3 + 3x^2 + 2$ at its point of inflection.

Answer: $(1, 3)$ and $(5, \infty)$

#_____ - Given $f(x) = \sqrt[3]{x}$. Write an equation of a tangent line to the function f when $x = 8$ and use it to approximate the value of $\sqrt[3]{11}$.

Answer: -3 and 3

#_____ - Evaluate: $\displaystyle \lim_{h \to 0} \dfrac{\sin\left(\dfrac{\pi}{3} + h\right) - \sin\dfrac{\pi}{3}}{h}$

Answer: $\dfrac{5}{2}$

#_____ - If $f(x) = \cos(4x)$, find $f'\left(\dfrac{\pi}{12}\right)$.

Answer: $y - 4 = -3(x+1)$

#_____ - A particle moves along a horizontal line so that its position is given by $s(t) = t^3 - 9t^2 + 15t + 4$, where $t \geq 0$. When is the particle moving to the left?
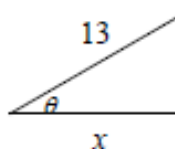
Answer: $-\dfrac{3}{2}$

#_____ - A spherical balloon is inflated with helium at the rate of $200\pi\,\dfrac{\text{ft}^3}{\text{min}}$. How fast is the balloon's radius increasing in ft/min at the instant the radius is 5 ft? $\left(\text{Volume of a sphere} = \dfrac{4}{3}\pi r^3\right)$

Answer: $\dfrac{2x\tan x - x^2\sec^2 x}{\tan^2 x}$

#_____ - Find y' given $y = \sin^4\left(x^3\right)$.

Answer: $\dfrac{1}{2}$

#_____ - If x is increasing at a rate of 2 units per second, find the rate of change of θ at the instant when $x = 12$ units.

Answer: $2\dfrac{1}{4}$

#_____ - Find two positive numbers such that the sum of the first plus two times the second is 100 and their product is a maximum.

Free Response – No Calculator

Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

(c) Show that if $f''(x) = 0$ for all x, then the graph of g does not have a point of inflection.

(d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

Use your calculator!

The second derivative of a function f is given by $f''(x) = \sin(3x) - \cos(x^2)$. How many points of inflection does the graph of f have on the interval $0 < x < 3$?

(A) One   (B) Three   (C) Four   (D) Five